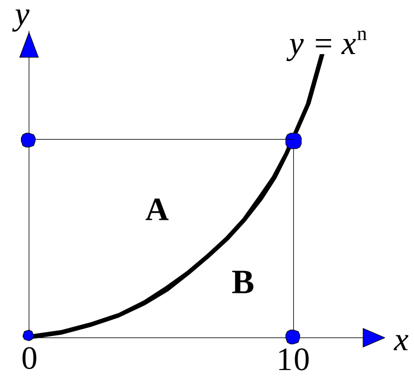
Power Maths

by Sidney Schuman

Go straight to the calculus power rules

Integral power rule



On the graph of $y = x^n$ a rectangle has been drawn from the point on the curve where $x = 10$. This rectangle is divided by the curve into two regions, A and B.
Let ten vertical strips of equal width fill the rectangle. The area 'below' the curve (B) can now be calculated using the mid-ordinate rule. Because the vertical strips are one unit wide, the approximate area of each vertical strip is simply the height of the mid-ordinate,

so $\mathrm{B} = 0.5^n + 1.5^n + 2.5^n + 3.5^n + 4.5^n + 5.5^n + 6.5^n + 7.5^n + 8.5^n + 9.5^n$

This can then be subtracted from the rectangle area to give the area 'above' the curve (A).

Calculate the area of regions A and B for n = 2, 3, 4 and 5; find the ratio of areas $\frac{\mathrm{A}}{\mathrm{B}}$ for each value of n

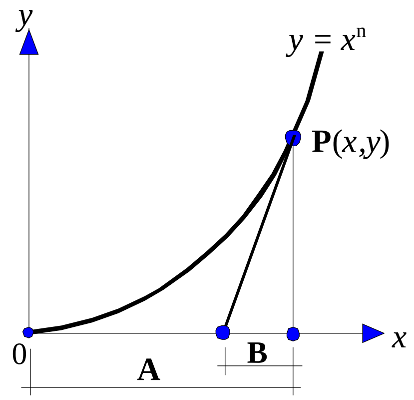
Your calculations should indicate that: $\frac{\mathrm{A}}{\mathrm{B}} = n$ *Equation 1*

Now, the area of the rectangle is xy but $y = x^n$ so the rectangle area is x^{n+1}

But the rectangle area is also the sum of the region areas A and B, so: $\mathrm{A} + \mathrm{B} = x^{n+1}$ *Equation 2*

Combine Equation 1 and Equation 2 to deduce the integral power rule for the area below the curve

Differential power rule



On the graph of $y = x^n$ two lines have been drawn from point $\mathrm{P}(x, y)$ on the curve.
One is perpendicular to the *x*-axis and the other is tangent to the curve, forming a right triangle. As the point moves round the curve, the tangent line varies in length and orientation. This movement contains information about the ratio of areas (above) which the tangent line projects on to the *x*-axis, resulting in a ratio of dimensions.

So again we have: $\frac{\mathrm{A}}{\mathrm{B}} = n$

Download and print graphs of $y = x^2$, $y = x^3$, $y = x^4$ and $y = x^5$. Draw tangent and ordinate from a point on each graph, measure dimensions A and B, check the ratio.

Given that $\mathrm{A} = x$ and $\frac{\mathrm{A}}{\mathrm{B}} = n$, we have: $\frac{x}{\mathrm{B}} = n$ *Equation 3*

Now, the rate of change $\frac{dy}{dx}$ of the curve $y = x^n$ is also the gradient (m) of the hypotenuse

But $\mathrm{m} = \frac{y}{\mathrm{B}}$ and since $y = x^n$ then $\mathrm{m} = \frac{x^n}{\mathrm{B}}$ *Equation 4*

Combine Equation 3 and Equation 4 to deduce the differential power rule for the rate of change of the curve